ATMOSPHERIC SCIENCE LETTERS
Atmos. Sci. Let. **14**: 41–44 (2013)
DOI: 10.1002/asl.414

A comparison of modern and historical methods for calculating Montgomery steamfunction

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Received: 12 April 2012
Revised: 12 July 2012
Accepted: 24 September 2012

Abstract

The suggestion has been made that the Montgomery streamfunction is now calculable for isentropic analysis purposes simply by applying the equation $\Psi = gZ + c_pT$. Historically, this practice has been frowned upon, as measurements aloft did not have the accuracy requisite to allow such practice. Instead, an integral approach to calculating Ψ has been preferred to get workable values. Yet, advancements in the accuracy of radiosonde measurements have occurred, especially with global positioning system (GPS)-verified height data. Herein, we show that the values from the simple and integral methods of getting Ψ are very similar, and differ by only $\sim$0.2%. Copyright © 2013 Royal Meteorological Society

Keywords: isentropic; Montgomery; streamfunction

1. Introduction

The calculation of the Montgomery streamfunction, Ψ, for the purposes of isentropic analysis has a long and turbulent history. Once the backbone of operational analysis before World War II (Moore, 1993), isentropic coordinates fell out of favor, in part, because of an inability to calculate Ψ (Reiter, 1972), and thus the geostrophic wind, reliably. This history is related by Bleck (1973) and Moore (1993), with detail on the problems with Ψ supplied by Danielsen (1959). Reiter (1972) expanded on Danielsen's (1959) work, and supplied detailed steps for how to integrate the equation for Ψ and stay beneath the error thresholds suggested by Danielsen (1959).

Yet, even as early as Danielsen's (1959) work it was understood and implied that the simple form for the Montgomery streamfunction is

$$\Psi = c_pT + gZ \tag{1}$$

where c_p is the specific heat of dry air, T the ambient air temperature, g the acceleration of gravity, and Z the geopotential height of an isentropic surface, would suffice, given accurate measurements of pressure and temperature. Recently, Lackmann (2011) has advocated just such an approach for calculating Ψ, suggesting that observed data and especially model output are sufficiently accurate so as to eliminate the need for an integral approach to Ψ.

This brief paper will use high-quality radiosonde data from several ascents over North America for calculations of Ψ using the simple method shown in Equation (1) as well as the method of Danielsen (1959).

2. Data and methods

For this study, data were collected with radiosonde flights using the University of Missouri (UM) sounding system, an International Met Systems iMet-3000 ground station, which is paired with iMet-1 AB radiosondes. The sondes allow for wind calculations using global positioning system (GPS) measurements, as well as GPS-verified heights. This is a system used commonly around the world (e.g. Cape Verde, Tanzania, Bangladesh, South Africa, Kwajalein Island) to collect operational data. The flights used in this study come from three locations around the Central United States in several seasons (Table I).

From these data, two calculations were made on the Montgomery streamfunction. The first is the simple approach, which was given in Equation (1). In that instance, the measured values for T and Z [the geometric height; transformed to Z using the $g = 9.80616\ \mathrm{m\,s^{-2}}$ common in the GEMPAK software (Koch *et al*., 1983), which we assume to be constant in the layer examined] are simply inserted into the expression to calculate Ψ. Recalculating g for each new level accounts for changes of less than 0.07% in the ratio of Danielsen and simple calculations for Ψ. In this work, calculations for each version of Ψ are done initially 30 s into the flight, and then every minute thereafter. When these data are processed into mandatory and significant levels, similar vertical resolutions result.

The second method is the integrated approach that follows Danielsen (1959; see Eqn. (6)) and has become the standard over the last few decades:

$$\Psi_{\theta_2} = \Psi_{\theta_1} + c_p\overline{T}\ln\left(\frac{\theta_2}{\theta_1}\right) \tag{2}$$

where Ψ_{θ_2} is the newly integrated Montgomery streamfunction value at the new upper level, Ψ_{θ_1} the Montgomery streamfunction value at the previous, lower level, $\overline{T}$ the mean temperature between those levels, and θ_2 and θ_1 are the potential temperatures at the upper and lower levels, respectively. Equation (2) is, in essence, a form of the hypsometric equation in isentropic space. The initial level is based upon observed temperature and the rawinsonde launch site's measured elevation above mean sea level. The first calculation is made 30 s into each flight; new calculations of Ψ_{θ_2} are then made every 60 s thereafter in the radiosonde ascent. For these calculations, 1-min averages of $\overline{T}$ are calculated for the layer between θ_2 and θ_1. The increment is calculated for that layer, and then added onto the previous Ψ_{θ_2}, which has become Ψ_{θ_1} in the new iteration.

3. Analysis

For the purposes of this paper, the accepted integral method for finding the Montgomery streamfunction (Equation (2)) is termed the accepted calculation, and the simple method (Equation (1)) is the experimental one (both calculations contain error), whose difference from the accepted value is also calculated and expressed here as:

$$\text{diff} = \left(1 - \frac{\Psi}{\Psi_{\theta_2}}\right) \times 100 \qquad (3)$$

Table I. Specifics on the radiosonde flights, the data from which the basis of this study form.

Date	Valid hour (UTC)	25-min Press. Elev. (hPa)	Location
13 May 2009	2000	503	Columbia, Missouri
9 December 2009	0400	450	Clinton, Iowa[a]
24 January 2010	1000	502	Whitewater, Wisconsin[a]
21 September 2010	2000	512	Columbia, Missouri
22 March 2011	1600	302	Columbia, Missouri
19 April 2011	1700	292	Columbia, Missouri

Date, launch hour (UTC), the pressure elevation (in hPa) at 25 min into the ascent, and the surface location of radiosonde release are detailed in the table.

[a] Collected during the Profiling of Winter Storms (PLOWS) project.

Only the first 25 min of each flight are examined, as revealed in both Table I and Figure 1. The 25-min time frame is an arbitrary choice for the purposes of this paper, as the difference profiles all exhibited the same profile throughout the rest of the tropopause.

As Table I shows, this choice of a data cutoff was sufficient to allow analysis of the lower and middle troposphere, and in a few flights, nearly to the tropopause level. Figure 1 shows that the difference profiles are largely unchanging with height; occasional aberrations appear as a result of warm or cold layers that can impact the local temperature calculations within a layer. Yet, even with these minor fluctuations, the calculated differences are well below the 2% threshold established by Danielsen (1959).

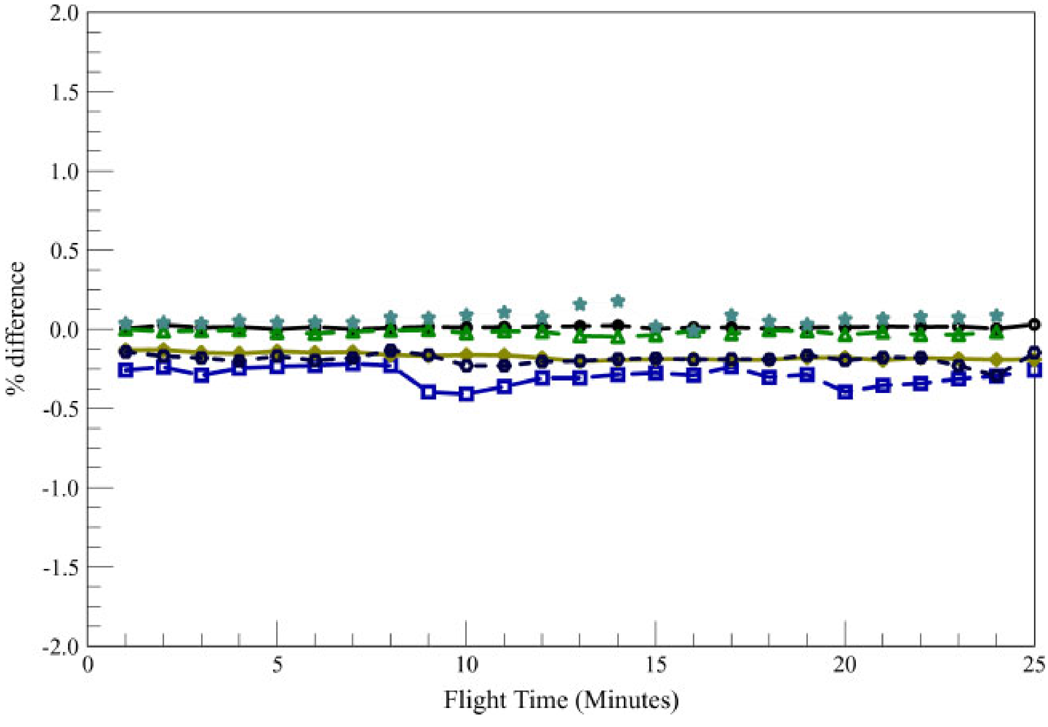
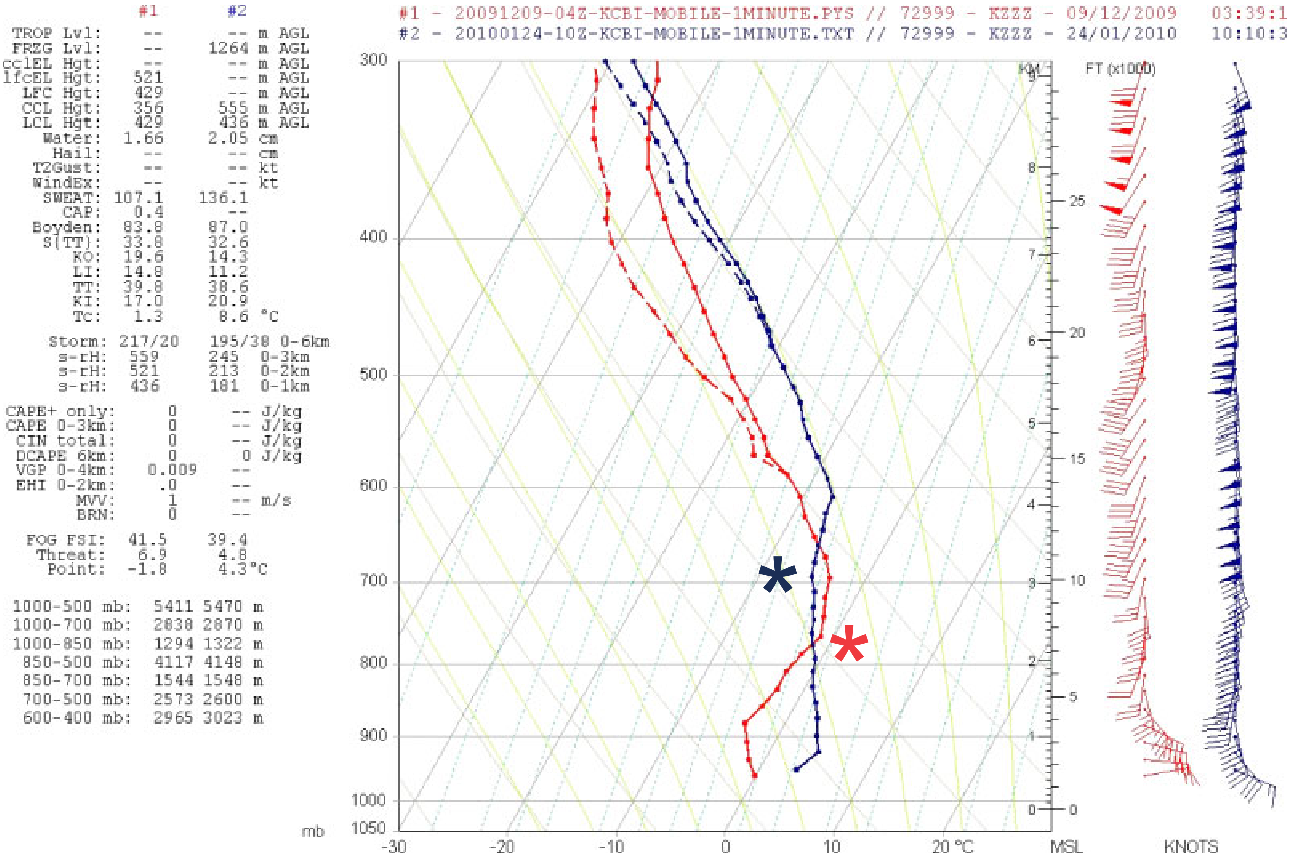


Figure 1. Calculated difference (in percent) between the integrated approach to Montgomery streamfunction (considered 'truth') and the simple method (considered the experimental) *versus* flight time (in minutes), for the six radiosonde flights listed in Table I. Each flight is marked uniquely: 13 May 2009 (diamonds), 9 December 2009 (squares), 24 January 2010 (stars), 21 September 2010 (hexagons), 22 March 2011 (circles), and 19 April 2011 (triangles).



```
              #1       #2
  TROP Lvl:    --       -- m AGL
  FRZG Lvl:    --     1264 m AGL
 cclEL Hgt:    --       -- m AGL
 lfcEL Hgt:   521       -- m AGL
   LFC Hgt:   429       -- m AGL
   CCL Hgt:   356      555 m AGL
   LCL Hgt:   429      436 m AGL
     Water:  1.66     2.05 cm
      Hail:    --       -- cm
    T2Gust:    --       -- kt
    WindEx:    --       -- kt
     SWEAT: 107.1    136.1
       CAP:   0.4       --
    Boyden:  83.8     87.0
     S(TT):  33.8     32.6
        KO:  19.6     14.3
        LI:  14.8     11.2
        TT:  39.8     38.6
        KI:  17.0     20.9
        Tc:   1.3      8.6 °C

     Storm: 217/20   195/38 0-6km
      s-rH:  559      245  0-3km
      s-rH:  521      213  0-2km
      s-rH:  436      181  0-1km

 CAPE+ only:      0      -- J/kg
 CAPE 0-3km:      0      -- J/kg
  CIN total:      0      -- J/kg
  DCAPE 6km:      0       0 J/kg
  VGP 0-4km:  0.009      --
  EHI 0-2km:     .0      --
        MVV:      1      -- m/s
        BRN:      0      --

   FOG FSI:  41.5     39.4
    Threat:   6.9      4.8
     Point:  -1.8      4.3°C

 1000-500 mb:  5411 5470 m
 1000-700 mb:  2838 2870 m
 1000-850 mb:  1294 1322 m
  850-500 mb:  4117 4148 m
  850-700 mb:  1544 1548 m
  700-500 mb:  2573 2600 m
  600-400 mb:  2965 3023 m
```

```
#1 - 20091209-04Z-KCBI-MOBILE-1MINUTE.PYS // 72999 - KZZZ - 09/12/2009   03:39:1
#2 - 20100124-10Z-KCBI-MOBILE-1MINUTE.TXT // 72999 - KZZZ - 24/01/2010   10:10:3
```

Figure 2. Vertical profiles of air temperature (solid; °C) dew point temperature (dashed; °C), and winds (knots, far right) plotted at 30 s into the flight (bottom-most data) and then every minute thereafter for 04:00 UTC 9 December 2009 (red) and 1000 UTC 24 January 2010 (blue), as in Figure 1. Red (blue) asterisk corresponds to times/levels of significant differences identified in Figure 1 for 8–9 min into the 04:00 UTC 9 December 2009 (13–14 min into the 1000 UTC 24 January 2010) sounding.

Maximum differences between the integral method (Equation (2)) and the simple method (Equation (1)) ranged from +0.19% (24 January 2010 flight) to −0.41% (9 December 2009 flight). The largest range of difference within a single flight also occurred on the 9 December 2009 ascent, with an inter-range value of 0.19%. Thus, most of the calculated differences at each of these levels are more than one order of magnitude smaller than what Danielsen (1959) showed to be an acceptable level of deviation.

The analysis in Figure 1 also shows that most of the difference calculations are negative, meaning that the difference (integral – simple) comes from the simple method calculation being slightly larger than that from the integral approach. This behavior arises from the simple method typically employing instantaneous temperatures that are slightly warmer (by at most ~0.2 °C) than the 1-min average temperature values used in the integral approach. There are also occasional aberrations in some of the traces in Figure 1, most notably for the soundings from 9 December 2009 (8–9 min into the flight) and 24 January 2010 (13–14 min into the flight). A closer examination of these soundings (Figure 2) reveals that these relatively large departures from the mean difference profile were due to this averaging approach in the vicinity of some changes in the lapse rate.

4. Summary and conclusions

Most meteorological data analysis and rendering software written in the last few decades have employed the integral approach of Danielsen (1959) to the calculation of the Montgomery streamfunction, and the success of these calculations cannot be overstated. Isentropic coordinates have now been employed for years with complete confidence. Yet, the quality of radiosonde observations has improved to the point that they have fulfilled Danielsen's (1959) prediction and Lackmann's (2011) assertion that the simpler method (Equation (1)) may well be used to calculate the Montgomery streamfunction with no ill effects. Our calculations show conclusively that the simple method for finding Ψ may be employed in the presence of highly accurate height and temperature data.

The implications of this finding do not have quite the reach of Danielsen's (1959), and may have less to do with encouraging broader use of isentropic analysis techniques and more to do with highlighting the marked improvement in radiosonde data. To the authors' knowledge, most existing software is written to compute Ψ with Danielsen's integral approach, and it is not expected that the existing code will be rewritten. However, we have now confirmed experimentally what had been discussed and speculated for some time, and asserted recently in print by Lackmann (2011), that radiosonde data have achieved a level of accuracy and precision to allow the calculation of Ψ with Equation (1).

Acknowledgements

We are indebted to two reviewers for their insightful comments, which improved the manuscript. Thanks go also to Mr Frederick Clowney of InterMet Systems, Inc. for sharing information on the global usage of their sounding systems, and to Gary Lackmann for discussions on this topic and his encouragement to explore this line of investigation. This work is supported, in part, by the National Science Foundation (NSF), Award No. ATM-0021656. Any opinions, findings, conclusions, or recommendations expressed herein are those of the authors and do not necessarily reflect the views of the NSF.

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